*Lesson №6. **One-dimensional movement in the piecewise continuous potentials. Discrete spectrum states, continuous spectrum states.***

1. Edge (boundary) conditions for piecewise continuous potentials:

1.1. «Infinite wall»

$$U(x) = \begin{cases} \infty, & x < x_0; \\ 0, & x > x_0. \end{cases} \qquad \psi(x \le x_0) = 0.$$
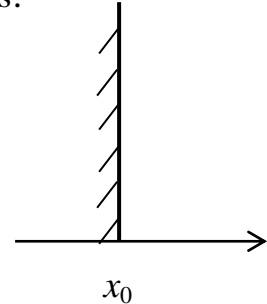
Wave function is always continuous, consequently,
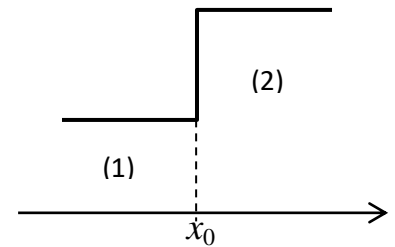
$$\psi(x_0) = 0.$$

1.2. Finite jump of potential energy

$$U(x) = \begin{cases} U_1, & x < x_0; \\ U_2, & x > x_0, \end{cases} \quad U_1 \ne U_2.$$

Wave function and it's first-order derivative are continious in the point x_0, consequently,

$$\psi_1(x_0) = \psi_2(x_0); \quad \psi_1'(x_0) = \psi_2'(x_0)$$
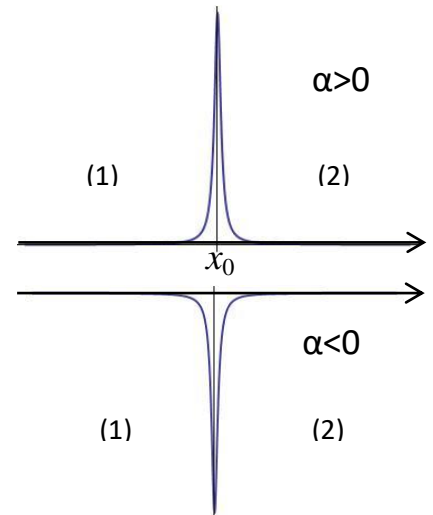
1.3. Delta-potential

$$U(x) = \alpha\delta(x - x_0)$$

Wave function is continuous in the point x_0, and first order derivative of wave function has a finite jump in the point x_0

$$\psi_2(x_0 + 0) = \psi_1(x_0 - 0);$$

$$\psi_2'(x_0 + 0) - \psi_1'(x_0 - 0) = \frac{2m\alpha}{\hbar^2}\psi(x_0).$$

2. The transmission coefficient and reflection coefficient

$$\vec{j} = \frac{\hbar}{2mi}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right); \quad D = \frac{\left|\vec{j}_{transmitted}\right|}{\left|\vec{j}_{incidental}\right|}; \quad R = \frac{\left|\vec{j}_{reflected}\right|}{\left|\vec{j}_{incidental}\right|}$$

Task 1. Investigate motion of the particle in rectangular potential well of an finite depth U_0.

$$U(x) = \begin{cases} -U_0, |x| < a, \\ 0, \ |x| > a. \end{cases}$$

Analyze the state of discrete (HKK № 2.7) and continuous spectrum (above barrier reflection, HKK № 2.49).

Task 2. Analyze motion of the particle in a field of a δ–well

$$U(x) = -\alpha\delta(x).$$

Analyze the state of discrete (HKK № 2.11) and continuous spectrum (above barrier reflection, HKK № 2.49)

Task 3. Find discrete energy spectrum for the particle in the field

$$U(x) = \begin{cases} \infty, x < 0, \\ -\alpha\delta(x-a), \ x > 0. \end{cases}$$

(HKK № 2.19).

Home task HKK № 2.17, 2.19, 2.43, 2.44, 2.46, 2.47, 2.48.

HKK- Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981
Hr. - Hrechko, Suhakov, Tomasevich, Fedorchenko Collection of theoretical physics problems, 1984